Temporal Instability Enables Neutrino Flavor Conversions Deep Inside Supernovae

Basudeb Dasgupta[1] and Alessandro Mirizzi[2]

[1] Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai, 400005, India.
[2] Dipartimento Interateneo di Fisica "Michelangelo Merlin," Via Amendola 173, 70126 Bari, Italy.
Istituto Nazionale di Fisica Nucleare - Sezione di Bari, Via Amendola 173, 70126 Bari, Italy.

(Dated: September 16, 2015)

We show that a self-interacting neutrino gas can spontaneously acquire a non-stationary pulsating component in its flavor content, with a frequency that can exactly cancel the "multi-angle" refractive effects of dense matter. This can then enable homogeneous and inhomogeneous flavor conversion instabilities to exist at large neutrino and matter densities, where the system would have been stable if the evolution were strictly stationary. Large flavor conversions very close to a supernova core may become possible via this novel mechanism, with important consequences for the explosion dynamics and nucleosynthesis, as well as for neutrino observations of supernovae.

PACS numbers: 14.60.Pq, 97.60.Bw

Introduction.— Collective flavor oscillations of neutrinos streaming off the core of a supernova (SN) are a rich and complex phenomenon. In the deepest SN regions, neutrino densities are so high that their flavor oscillations are affected not only by ordinary matter, but also by the neutrinos themselves [1–4]. Neutrino-neutrino interactions lead to highly correlated flavor conversions and many unexpected effects, which can completely change the physics of supernova neutrinos [5–23]. See refs. [24, 25] for recent reviews.

Above the neutrinosphere, in the absence of collisions, the dynamics of a dense neutrino gas is characterized in terms of "density matrices" in flavor space, $\varrho(t, \mathbf{x}, E, \mathbf{v})$ for neutrinos with energy E and velocity $\mathbf{v}$ at position $\mathbf{x}$ and time t, which obey kinetic equations [21, 26, 27],

$$i(\partial_t + \mathbf{v}\cdot\nabla)\varrho = [\mathsf{H}, \varrho] \ . \tag{1}$$

On the left-hand-side (l.h.s.), the first term accounts for an explicit time dependence, while the second, proportional to the neutrino velocity $\mathbf{v}$, is the drift-term due to neutrino free-streaming. On the right-hand-side (r.h.s.), $\mathsf{H}(t, \mathbf{x}, E, \mathbf{v})$ is the Hamiltonian matrix in flavor space containing the neutrino mass-square matrix and potentials due to matter and neutrinos.

Flavor evolution of a dense neutrino gas, which is governed by eq. (1), has a highly complex structure. It depends on the 4 time and space coordinates, the 4 energy and velocity coordinates (with $|\mathbf{v}| = 1$, in our ultra-relativistic approximation), as well as the flavor states of all neutrinos. In order to reduce this complexity, symmetries in the neutrino flavor evolution have often been assumed. For neutrinos in a SN environment, all of previous literature is based on the assumption that the evolution is stationary, i.e., there is no explicit time-dependence, or only a slow/small time-dependence which does not affect flavor evolution significantly. Additionally, under the assumption of a spherically symmetric neutrino emission, the dynamics reduces to a one-dimensional evolution along the radial coordinate. This is the rationale behind the often-used "bulb model" [5, 7].

These symmetry assumptions, viz., temporal stationarity and spatial homogeneity, have been recently criticized because self-interacting neutrinos can spontaneously break these space-time symmetries. Indeed, studies on simple toy-models show that the translation symmetries in time [28, 29] and space [30–34] are not stable. Even tiny space inhomogeneities may lead to new flavor instabilities [30–32], which can develop even at small distances from the SN core, i.e., at large neutrino densities, where oscillations are otherwise expected to be suppressed due to synchronization [30]. However, large neutrino densities in a SN are typically accompanied by a large matter density [34], which produces "multi-angle matter effects" [35] that suppress both homogeneous and inhomogeneous instabilities. The current understanding is then that neutrinos cannot change their flavor too close to the SN core.

Flavor conversions at small distances from the SN core would have major consequences for SN explosions, nucleosynthesis, as well as neutrino observations of nearby SNe. If conversions are possible below the shock radius, neutrinos can provide a net positive energy to the shock and assist SN explosions [36–39]. Similarly, the neutron-to-proton ratio can be changed deeper inside a star, affecting the yield of heavier nuclei created through the r-process [40, 41]. Also, in order to interpret any potential observation of neutrinos from SNe, current and proposed neutrino experiments depend crucially on understanding how and where the flavor-dependent neutrino fluxes have converted to each other [25, 42–44]. Now that Gd-doping in Super-Kamiokande [45] is approved [46], the imminent observation of the diffuse background of SN neutrinos may raise this issue [47] sooner than a Galactic SN observation.

In this Letter, we point out *pulsating instabilities*, that may lead to flavor conversions at high neutrino and matter densities. The key insight is that the frequency of pulsation can undo the phase dispersion due to a large matter density. As a result, flavor instabilities, that would grow only if matter effects were small, can now develop at large neutrino and matter densities.

In the following, first we provide an analytical argument, using linear stability analysis, to show the presence of a pulsating mode and explain why it is unstable. Then, to demonstrate that this linear instability survives in the non-linear regime, we numerically calculate the flavor evolution in a simplified model with two neutrino beams, and show that flavor conversions occurs at large neutrino and matter densities. Finally, we discuss the implications for SN neutrinos and conclude.

Linear analysis for a general scenario.— Assuming that the neutrinos are initially in flavor eigenstates, their density matrices $\varrho(t, \mathbf{x}, E, \mathbf{v})$ can be written in a 2-flavor framework as

$$\varrho = \frac{\mathrm{Tr}(\varrho)}{2} + \frac{n_\nu}{2} g \begin{pmatrix} 1 & S \\ S^* & -1 \end{pmatrix}, \tag{2}$$

to linear order in $S(t, \mathbf{x}, E, \mathbf{v})$ [48]. The quantity $g(t, \mathbf{x}, E, \mathbf{v})$ is the energy and angular distribution of neutrinos from the source and n_ν is an arbitrary normalization constant, with dimensions of number density, for making S dimensionless. A non-zero off-diagonal element S represents flavor conversions. For antineutrinos, $\bar{\varrho}(E) \equiv -\varrho(-E)$, extending the physical range of E from $-\infty$ to $+\infty$. The Hamiltonian for the flavor evolution is

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2} G_F \mathsf{N}_l + \sqrt{2} G_F \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') \varrho', \tag{3}$$

where sans-serif quantities are 2×2 matrices in flavor space. Namely, M^2 is the neutrino mass-squared matrix, $\sqrt{2} G_F \mathsf{N}_l$ appears due to forward scattering on matter [49], and the last term on the r.h.s. encodes refractive effects of interactions among neutrinos [1, 2]. The integral is over all neutrino energies and velocities, i.e., $\int d\Gamma' = \int_{-\infty}^{+\infty} dE'\, E'^2 \int d\mathbf{v}'/(2\pi)^3$.

In a non-isotropic neutrino gas, as in the case of neutrinos streaming off a SN core, there is a net neutrino current so that neutrinos moving in different directions, i.e., with different $\mathbf{v}$, acquire different phases via the velocity-dependent terms, i.e., $(1 - \mathbf{v} \cdot \mathbf{v}')$ in H and $\mathbf{v} \cdot \nabla$ from the drift-term in eq. (1). These are multi-angle effects, that arise due to the current-current nature of the low-energy weak interactions and the source geometry. In some situations, they inhibit the collective behavior of the flavor evolution observed in an isotropic case, sometimes leading to flavor decoherence [50–52].

Using eqs. (2) and (3) in eq. (1), and taking a vanishing mixing angle, we find the equation for flavor evolution,

$$\begin{aligned} i(\partial_t + \mathbf{v} \cdot \nabla) S = \big[\omega + \lambda + \mu {\textstyle\int} d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') g'\big] S \\ - \mu {\textstyle\int} d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') g' S', \end{aligned} \tag{4}$$

where the relevant energy scales are the neutrino oscillation frequency in vacuum $\omega = \Delta m^2/(2E)$, the matter potential $\lambda = \sqrt{2} G_F n_e$, and the neutrino potential $\mu = \sqrt{2} G_F\, n_\nu$. Note that μ always appears in product with S, making the precise choice of n_ν immaterial.

Let us consider the evolution of S along the radial distance r, while Fourier decomposing it in t and the spatial coordinates transverse to $\hat{r}$, viz., $\mathbf{r}_T$. We will take the spectrum $g'(t, \mathbf{x}, E, \mathbf{v})$ to be independent of time and space, i.e., $g'(t, \mathbf{x}, E, \mathbf{v}) \equiv g'(E, \mathbf{v})$, so that it does not get Fourier transformed. Explicitly,

$$S = \int_{-\infty}^{+\infty} dp\, d\mathbf{k}\, e^{-i(pt + \mathbf{k} \cdot \mathbf{r}_T)} Q_{p,\mathbf{k}} e^{-i\Omega\, r}, \tag{5}$$

where $Q_{p,\mathbf{k}} e^{-i\Omega\, r}$ is the Fourier coefficent of a flavor evolution mode with temporal pulsation p and inhomogeneity wavevector $\mathbf{k}$. Inserting this ansatz into eq. (4), using $\mathbf{v} \cdot \nabla = v_r \partial_r + \mathbf{v}_T \cdot \nabla_T$, and dividing by the radial velocity v_r, we find an eigenvalue equation for $Q_{p,\mathbf{k}}(E, \mathbf{v})$,

$$\begin{aligned} \left[\frac{\omega + \bar{\lambda} - p - \mathbf{v}_T \cdot \mathbf{k}}{v_r} - \Omega\right] Q_{p,\mathbf{k}} = \\ \frac{\mu}{v_r} \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') g' Q'_{p,\mathbf{k}}, \end{aligned} \tag{6}$$

where $\bar{\lambda} = \lambda + \mu \int d\Gamma' (1 - \mathbf{v} \cdot \mathbf{v}') g'$ encodes "matter" effects from both matter and neutrinos. Note that in the linear regime, different Fourier modes are not coupled. A growing solution to this equation, with $\mathrm{Im}(\Omega) > 0$, signals that there is an instability.

For a stationary system one finds such growing solutions even at a large neutrino density, if inhomogeneities are present, i.e., $\mathbf{k} \neq 0$, as long as $\bar{\lambda} \ll \mu$ [30, 31, 34]. However, these instabilities are typically not realized for SNe, where $\bar{\lambda}$ is also large when μ is large [34]. The reason for this is clear: One cannot simultaneously satisfy the eigenvalue equation for all velocities, because the inhomogeneous term, the matter term, and the μ-dependent neutrino-neutrino interaction term on the r.h.s. are all large, i.e., $\mathbf{v}_T \cdot \mathbf{k}$, $\bar{\lambda}$, $\mu \gg \omega$, but have different velocity-dependences which cannot completely cancel against each other.

The non-stationary system has an innocuous-looking but important difference with respect to the stationary system. Non-stationarity lowers $\bar{\lambda}$ by p, i.e., $\bar{\lambda} \to \bar{\lambda} - p$. This was also pointed out in ref. [29], which appeared while our manuscript was in its final stages. Importantly, this $p \neq 0$ term has the same velocity-dependence as $\bar{\lambda}$. Therefore, if one allows for a non-stationary solution, the neutrino system with a pulsation $p \simeq \bar{\lambda}$ can undo the phase dispersion due to a large matter term for all velocities. Thereafter, one can find growing solutions, with $\mathrm{Im}(\Omega) > 0$, to the eigenvalue equation [eq. (6)], as previously for the $\bar{\lambda} \ll \mu$ scenario. These solutions are highly oscillatory in space ($\mathbf{k} \neq 0$) and time ($p \neq 0$), and would lead to flavor averaging. *This is our main result.* The eigenvalues are identical to those in Sec. 4.2.4 of ref. [34] with the shift $\bar{\lambda} \to \bar{\lambda} - p$.

We show this main point in Fig. 1. The thick curves schematically show regions of instability where certain Fourier modes with pulsation p and wavevector $\mathbf{k}$ are unstable, i.e., have $\mathrm{Im}(\Omega) > 0$, relative to a SN matter

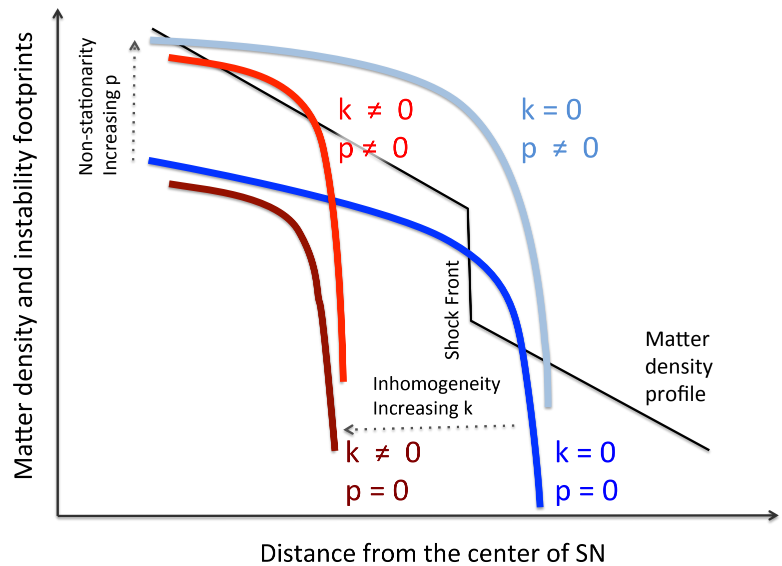

Figure 1: Schematic of regions of flavor instability, where $\mathrm{Im}(\Omega) > 0$ solutions to eq. (6) exist, relative to the matter potential λ, for zero and non-zero spatial inhomogeneity wavenumber k and frequency of temporal non-stationarity p.

density profile (thin line). The $\mathbf{k} \neq 0$ instability is always below the $\mathbf{k} = 0$ instability [34], and never occurs for the physically available matter density. However, $p \gtrsim \bar{\lambda}$ can *raise* both the $\mathbf{k} = 0$ and $\mathbf{k} \neq 0$ instabilities, making them unstable at low radii. These instabilities, especially those with a large wavenumber $k \equiv |\mathbf{k}|$, can now develop at large μ and λ, i.e., at a small radius, with a temporal oscillation of frequency p. This can happen for both normal and inverted neutrino mass ordering.

How does this linear instability evolve when linear theory is no longer appropriate? What is its impact? To answer these questions, we must numerically solve the equations of motion (EoMs), eq. (1), in the fully non-linear regime. This is what we do next.

Non-linear analysis for the two-beams model.— The new effect, relevant to SN neutrinos, is that a stationary system, which is stable to both homogeneous and inhomogeneous perturbations at large μ and λ, becomes unstable therein when non-stationarity is allowed. This requires simulating a system with temporal non-stationarity, spatial inhomogeneity, and multi-angle matter effects, which is extremely challenging and has not been attempted so far. Here, we present the first simulation with these three features.

The simplest model that can accommodate the required features is the neutrino "line model" [30, 32, 34]. In this model, one considers monochromatic neutrinos emitted in two directions, "L" and "R", from an infinite plane at $z = 0$. Assuming translational invariance along the y-direction, the flavor evolution along $z > 0$ can be characterized on the two-dimensional plane spanned by the x and z coordinates. The neutrino emission modes L and R are labeled in terms of their velocities, i.e., $\mathbf{v}_L = (v_{x,L}, 0, v_{z,L}) = (\cos\vartheta_L, 0, \sin\vartheta_L)$, where $\vartheta_L \in [0, \pi]$ is the emission angle, and similarly for $\mathbf{v}_R$. Thus, for the L mode, the differential operator on the l.h.s. in eq. (1) takes the form

$$\partial_t + \mathbf{v}_L \cdot \nabla = \partial_t + v_{x,L}\partial_x + v_{z,L}\partial_z \,, \tag{7}$$

while the Hamiltonian in eq. (3) becomes

$$\mathsf{H}_L = \frac{\omega + \lambda}{2}\sigma_3 + \mu(1 - \mathbf{v}_L \cdot \mathbf{v}_R)\left[(1+\epsilon)\varrho_R - \bar{\varrho}_R\right] \,, \tag{8}$$

where σ_3 is the diagonal Pauli matrix, and ϵ is the neutrino-antineutrino asymmetry, i.e., $1 + \epsilon = (n_{\nu_e} - n_{\nu_x})/(n_{\bar{\nu}_e} - n_{\bar{\nu}_x})$, with ν_x being a non-electron flavor. The normalization $n_\nu = n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$ is used to define $\mu = \sqrt{2} G_F\, (n_{\bar{\nu}_e} - n_{\bar{\nu}_x})$.

The differential operator in eq. (7) shows that the flavor evolution is determined by a partial differential equation in one temporal and two spatial dimensions. By Fourier transforming the EoMs in t and x, as in eq. (5), one obtains a tower of ordinary differential equations in the z-coordinate for the different Fourier modes $\varrho_{p,k}$ with temporal pulsation p and spatial wavenumber k. In the non-linear regime, the EoMs for the different Fourier modes have a convolution term due to interactions between the different modes [32].

If $v_{z,L} = v_{z,R}$, as assumed in refs. [30, 32], even a very large matter term λ can be rotated away from the EoMs by studying the flavor evolution in a suitable co-rotating frame [6]. Conversely, if $v_{z,L} \neq v_{z,R}$ the matter term leads to frequencies $\lambda/v_{z,L}$ for the L mode and $\lambda/v_{z,R}$ for the R mode, and their difference cannot be removed, producing multi-angle matter effects [35]. If $\lambda \gg \mu$, the phase difference between L and R modes is so large that it suppresses the self-induced flavor conversions from both $k = 0$ and $k \neq 0$ instabilities [34, 35]. Conversely, if one

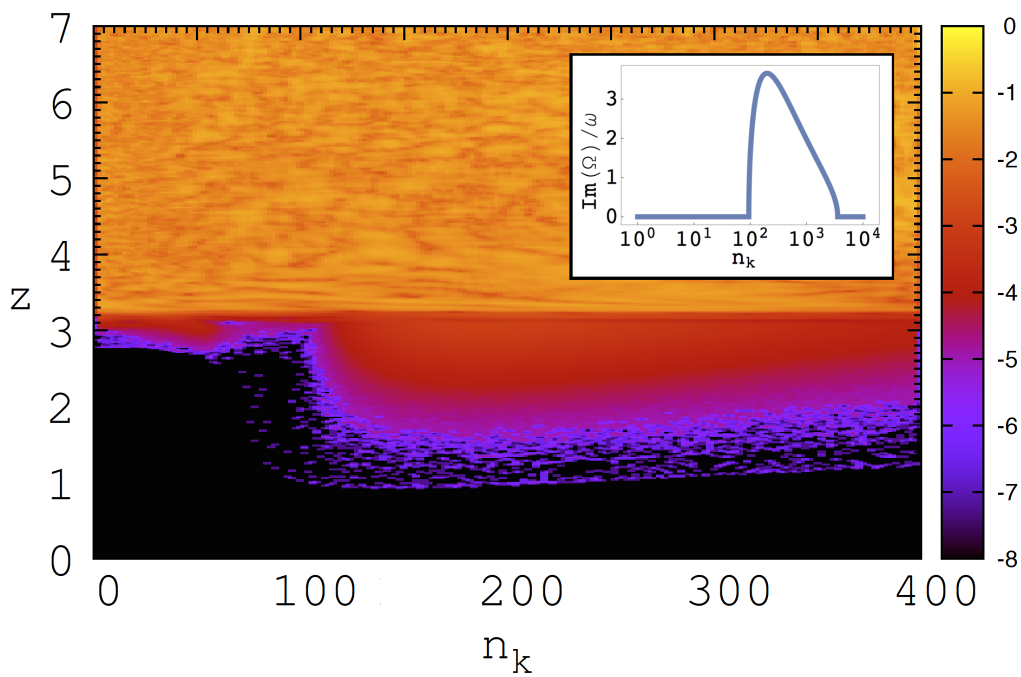

Figure 2: Non-linearly computed amplitude of spatially inhomogeneous flavor conversion at distance z, i.e., $\log_{10}|\varrho^{e\mu}_{n_k}|$, for a range of modes n_k in the presence of large neutrino and matter density, as well as non-stationarity ($n_p = 1$) in a two-beams model. Inset: Linear growth rates $\mathrm{Im}(\Omega)/\omega$ for a larger range of n_k (note the log-scale). See text for details.

allows for a non-stationary solution, the neutrino system selects the pulsation $p \simeq \bar{\lambda}$, that compensates the phase dispersion due to a large matter term, and generates growing instabilities, in particular at small-scales associated with spatial inhomogeneities.

To quantitatively illustrate this claim, we take the source at $z = 0$ to emit only ν_e and $\bar{\nu}_e$, with an factor of two excess of ν_e over $\bar{\nu}_e$, i.e., $\epsilon = 1$. The overall frequency-scale is set by $\omega = 1$. A large $\mu = 40$ is chosen, so that oscillations are suppressed in the homogeneous case, as in a SN. We take the L and R modes to have two different angles $\vartheta_R = 5\pi/18$ and $\vartheta_L = 7\pi/9$, so that a large matter density, $\lambda = 4 \times 10^4$, suppresses the inhomogeneous modes, mimicking a SN. In order to make k and p dimensionless, k is expressed in multiples of the vacuum oscillation frequency, i.e., $k = n_k\,\omega$, while p is expressed in multiples of the matter potential $\bar{\lambda}$, i.e., $p = n_p\,\bar{\lambda}$. For simplicity, we limit ourselves to the $n_p = 1$ mode. In this way, in the non-linear regime we only have to consider the convolution among the different Fourier modes associated with spatial inhomogeneities

$$\sim \sum_{j_k} \left[\left((1+\epsilon)\varrho_{R,n_k-j_k} - \bar{\varrho}_{R,n_k-j_k} \right), \varrho_{L,j_k} \right] , \qquad (9)$$

and analogously for the R mode [32]. To seed the spatial inhomogeneity, we use numerical noise of $\mathcal{O}(10^{-8})$ for all the modes. We choose $\theta = 10^{-3}$ and a normal mass ordering, i.e., $\omega > 0$, but the result would be similar for the inverted ordering, i.e., $\omega < 0$.

In Fig. 2, we show the flavor dynamics of this system through a contour plot of the Fourier amplitudes $\log_{10}|\varrho^{e\mu}_{n_k}|$ for various n_k at increasing distances z. We include up to the $n_k = 600$ mode, but impose that the modes with $n_k > 400$ remain empty. This well-known trick is adopted in order to avoid "spectral blocking" or recurrence effects that lead to a spurious rise of the Fourier coefficients at large n_k due to truncation of the tower of equations [53]. The Fourier modes start to grow at $z \gtrsim 1$. The homogeneous mode ($n_k = 0$) is stable for the large flavor asymmetry chosen here and the modes around $n_k \simeq 100$ grow first, because they are most unstable. This is consistent with the linear stability estimate for the most unstable modes, $n_k \simeq \mathcal{O}(100)$, as shown in Fig. 2 (inset). The cascade in Fourier space, initiated at $n_k \simeq 100$, then develops to both smaller and larger scales due to the convolution enforced by the non-linear interaction term [eq. (9)]. The instabilities initially grow approximately an order of magnitude for a unit step in z, consistent with the predicted growth rate $\mathrm{Im}(\Omega)/\omega \simeq 3$. If non-stationarity is switched off, i.e., $p = 0$, these instabilities do not grow.

Thus, we find that neutrinos can change flavor at large λ and μ, if non-stationary solutions with frequency $p \gtrsim \lambda$ are allowed. We expect that the cascade in Fourier space leads to "flavor decoherence", or approximate equilibration between all flavors [54]. In a SN, this may have important consequences, which we discuss below.

Discussion and conclusions.— An important question is if this effect can be important in a SN. Here, we provide a back-of-the-envelope estimate. If flavor instability has to occur well below the shock-front at $r \simeq 200\,\mathrm{km}$, and just above the gain radius at $r \simeq 100\,\mathrm{km}$ (see, e.g., Fig. 4 in ref. [38]), one needs pulsations of high frequency $p \gtrsim \lambda \sim 10^6\,\mathrm{km}^{-1} \sim 3 \times 10^{11}\,\mathrm{Hz}$ or larger. However, between $r \simeq 100\,\mathrm{km}$ to 150 km, a typical instability with growth rate $\mathrm{Im}(\Omega)/\omega \simeq 3$ grows by ~ 60 e-foldings for 15 MeV neutrinos with $\omega \sim 0.4\ \mathrm{km}^{-1}$. So initial seeds as small as $\mathcal{O}(10^{-26})$ can become fully non-linear. In a SN, can such high-frequency fluctuations occur with even this tiny amplitude? In this context, we find it intriguing that pair-correlations of the neutrino field, which are many-body corrections to the single-particle density matrices, lead to relative number fluctuations of a size $\kappa^2 \sim (\lambda\beta/E)^2 \sim 10^{-22}$, where $\beta \simeq 10^{-2}c$ is the typical speed of ordinary matter in SN, which oscillate with a frequency $\sim 2E \sim 10^{22}$Hz for 15 MeV neutrinos, as shown in ref. [23].

Potentially, the consequences of our finding augur another paradigm-shift in the understanding of self-induced conversions and on their impact on the SN dynamics. The possibility of low-radii conversions behind the stalled shock-wave during the accretion phase, suppressed by the large matter term in the stationary and homogeneous case [55–59], implies that the flavor dynamics may need to be taken into account in the revitalization of the shock-wave [36, 38]. Also, the impact on nucleosynthesis in a SN would be important [40, 41]. With flavor equilibration, the interpretation of observed SN fluxes may also become simpler [54].

However, the possibility of flavor conversions close to the neutrinosphere in a SN also questions the assumption that flavor conversions safely occur outside it. This assumption allowed one to replace the full Boltzmann equations, containing both oscillations and scatterings, with the flavor oscillation equations for free-streaming neutrinos. In a non-stationary situation, this assumption may no longer be guaranteed and imply the necessity to simultaneously perform the neutrino transport and flavor evolution [21]. This is a formidable problem that would require new computational techniques. Furthermore, it is possible that these instabilities (inhomogeneity and non-stationarity) appear in a regime where the coarse-grained description adopted using density matrices [2] is insufficient. Although we have used this standard description here, as a first step, this is a more fundamental aspect that needs further studies.

In conclusion, we have presented the first study of non-linear effects of non-stationarity in a dense neutrino gas. We have pointed out novel temporal instabilities that can dramatically affect flavor evolution, and raise the possibility of self-induced flavor conversions deep in a SN. The discovery of the role of symmetry-breaking in the flavor evolution of SN neutrinos is opening completely new directions of investigations. We foresee that many surprises are still in store.

Acknowledgments.— We warmly thank A. Dighe, S. Shalgar, and H. Vogel for useful discussions, and especially E. Lisi, G. Raffelt, and A. Smirnov for a careful reading of the manuscript and useful comments to improve it. B.D. thanks the organizers of TAUP 2015 at Torino, whose invitation motivated this work. The work of A.M. is supported by the Italian Ministero dell'Istruzione, Università e Ricerca (MIUR) and Istituto Nazionale di Fisica Nucleare (INFN) through the "Theoretical Astroparticle Physics" project.

[1] J. T. Pantaleone, "Dirac neutrinos in dense matter," Phys. Rev. D **46**, 510 (1992).

[2] J. T. Pantaleone, "Neutrino flavor evolution near a supernova's core," Phys. Lett. B **342**, 250 (1995) [astro-ph/9405008].

[3] Y. Z. Qian and G. M. Fuller, "Neutrino-neutrino scattering and matter enhanced neutrino flavor transformation in Supernovae," Phys. Rev. D **51**, 1479 (1995) [astro-ph/9406073].

[4] S. Pastor and G. Raffelt, "Flavor oscillations in the supernova hot bubble region: Nonlinear effects of neutrino background," Phys. Rev. Lett. **89** (2002) 191101 [astro-ph/0207281].

[5] H. Duan, G. M. Fuller, J. Carlson and Y. -Z. Qian, "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories," Phys. Rev. D **74**, 105014 (2006) [astro-ph/0606616].

[6] S. Hannestad, G. G. Raffelt, G. Sigl and Y. Y. Y. Wong, "Self-induced conversion in dense neutrino gases: Pendulum in flavour space," Phys. Rev. D **74**, 105010 (2006) [Erratum-ibid. D **76**, 029901 (2007)] [astro-ph/0608695].

[7] G. L. Fogli, E. Lisi, A. Marrone and A. Mirizzi, "Collective neutrino flavor transitions in supernovae and the role of trajectory averaging," JCAP **0712**, 010 (2007) [arXiv:0707.1998 [hep-ph]].

[8] G. L. Fogli, E. Lisi, A. Marrone, A. Mirizzi and I. Tamborra, "Low-energy spectral features of supernova (anti)neutrinos in inverted hierarchy," Phys. Rev. D **78**, 097301 (2008) [arXiv:0808.0807 [hep-ph]].

[9] B. Dasgupta and A. Dighe, "Collective three-flavor oscillations of supernova neutrinos," Phys. Rev. D **77**, 113002 (2008) [arXiv:0712.3798 [hep-ph]].

[10] B. Dasgupta, A. Dighe, G. G. Raffelt and A. Y. Smirnov, "Multiple Spectral Splits of Supernova Neutrinos," Phys. Rev. Lett. **103** (2009) 051105 [arXiv:0904.3542 [hep-ph]].

[11] A. Friedland, "Self-refraction of supernova neutrinos: mixed spectra and three-flavor instabilities," Phys. Rev. Lett. **104**, 191102 (2010) [arXiv:1001.0996 [hep-ph]].

[12] B. Dasgupta, G. G. Raffelt and I. Tamborra, "Triggering collective oscillations by three-flavor effects," Phys. Rev. D **81** (2010) 073004 [arXiv:1001.5396 [hep-ph]].

[13] B. Dasgupta, A. Mirizzi, I. Tamborra and R. Tomàs, "Neutrino mass hierarchy and three-flavor spectral splits

of supernova neutrinos," Phys. Rev. D **81**, 093008 (2010) [arXiv:1002.2943 [hep-ph]].

[14] J. F. Cherry, J. Carlson, A. Friedland, G. M. Fuller and A. Vlasenko, "Neutrino scattering and flavor transformation in supernovae," Phys. Rev. Lett. **108**, 261104 (2012) [arXiv:1203.1607 [hep-ph]].

[15] G. Raffelt, S. Sarikas and D. de Sousa Seixas, "Axial Symmetry Breaking in Self-Induced Flavor Conversion of Supernova Neutrino Fluxes," Phys. Rev. Lett. **111**, no. 9, 091101 (2013) [Phys. Rev. Lett. **113**, no. 23, 239903 (2014)] [arXiv:1305.7140 [hep-ph]].

[16] A. Mirizzi, "Multi-azimuthal-angle effects in self-induced supernova neutrino flavor conversions without axial symmetry," Phys. Rev. D **88**, no. 7, 073004 (2013) [arXiv:1308.1402 [hep-ph]].

[17] S. Chakraborty and A. Mirizzi, "Multi-azimuthal-angle instability for different supernova neutrino fluxes," Phys. Rev. D **90**, no. 3, 033004 (2014) [arXiv:1308.5255 [hep-ph]].

[18] H. Duan, "Flavor Oscillation Modes In Dense Neutrino Media," Phys. Rev. D **88**, 125008 (2013) [arXiv:1309.7377 [hep-ph]].

[19] Y. Pehlivan, A. B. Balantekin, T. Kajino and T. Yoshida, "Invariants of Collective Neutrino Oscillations," Phys. Rev. D **84** (2011) 065008 [arXiv:1105.1182 [astro-ph.CO]].

[20] C. Volpe, D. Väänänen and C. Espinoza, "Extended evolution equations for neutrino propagation in astrophysical and cosmological environments," Phys. Rev. D **87** (2013) 11, 113010 [arXiv:1302.2374 [hep-ph]].

[21] A. Vlasenko, G. M. Fuller and V. Cirigliano, "Neutrino Quantum Kinetics," Phys. Rev. D **89**, no. 10, 105004 (2014) [arXiv:1309.2628 [hep-ph]].

[22] J. Serreau and C. Volpe, "Neutrino-antineutrino correlations in dense anisotropic media," Phys. Rev. D **90**, no. 12, 125040 (2014) [arXiv:1409.3591 [hep-ph]].

[23] A. Kartavtsev, G. Raffelt and H. Vogel, "Neutrino propagation in media: Flavor-, helicity-, and pair correlations," Phys. Rev. D **91** (2015) 12, 125020 [arXiv:1504.03230 [hep-ph]].

[24] H. Duan, G. M. Fuller and Y. -Z. Qian, "Collective Neutrino Oscillations," Ann. Rev. Nucl. Part. Sci. **60**, 569 (2010) [arXiv:1001.2799 [hep-ph]].

[25] A. Mirizzi, I. Tamborra, H. T. Janka, N. Saviano, K. Scholberg, R. Bollig, L. Hudepohl and S. Chakraborty, "Supernova Neutrinos: Production, Oscillations and Detection," arXiv:1508.00785 [astro-ph.HE].

[26] G. Sigl and G. Raffelt, "General kinetic description of relativistic mixed neutrinos," Nucl. Phys. B **406**, 423 (1993).

[27] P. Strack and A. Burrows, "Generalized Boltzmann formalism for oscillating neutrinos," Phys. Rev. D **71**, 093004 (2005) [hep-ph/0504035].

[28] G. Mangano, A. Mirizzi and N. Saviano, "Damping the neutrino flavor pendulum by breaking homogeneity," Phys. Rev. D **89**, no. 7, 073017 (2014) [arXiv:1403.1892 [hep-ph]].

[29] S. Abbar and H. Duan, "Neutrino flavor instabilities in a time-dependent supernova model," arXiv:1509.01538 [astro-ph.HE].

[30] H. Duan and S. Shalgar, "Flavor instabilities in the neutrino line model," Phys. Lett. B **747**, 139 (2015) [arXiv:1412.7097 [hep-ph]].

[31] S. Abbar, H. Duan and S. Shalgar, "Flavor instabilities in the multi-angle neutrino line model," arXiv:1507.08992 [hep-ph].

[32] A. Mirizzi, G. Mangano and N. Saviano, "Self-induced flavor instabilities of a dense neutrino stream in a two-dimensional model," Phys. Rev. D **92**, no. 2, 021702 (2015) [arXiv:1503.03485 [hep-ph]].

[33] A. Mirizzi, "Breaking the symmetries of the bulb model in two-dimensional self-induced supernova neutrino flavor conversions," arXiv:1506.06805 [hep-ph].

[34] S. Chakraborty, R. S. Hansen, I. Izaguirre and G. Raffelt, "Self-induced flavor conversion of supernova neutrinos on small scales," arXiv:1507.07569 [hep-ph].

[35] A. Esteban-Pretel, A. Mirizzi, S. Pastor, R. Tomàs, G. G. Raffelt, P. D. Serpico and G. Sigl, "Role of dense matter in collective supernova neutrino transformations," Phys. Rev. D **78**, 085012 (2008) [arXiv:0807.0659 [astro-ph]].

[36] G. M. Fuller, R. Mayle, S. Bradley Meyer, J.R. Wilson, "Can a closure mass neutrino help solve the supernova shock reheating problem?," Astrophys. Journ. Part 1, vol. 389, p. 517-526 (1992).

[37] Y. Suwa, K. Kotake, T. Takiwaki, M. Liebendorfer and K. Sato, "Impacts of collective neutrino oscillations on core-collapse supernova explosions," Astrophys. J. **738** (2011) 165 [arXiv:1106.5487 [astro-ph.HE]].

[38] B. Dasgupta, E. P. O'Connor and C. D. Ott, "The Role of Collective Neutrino Flavor Oscillations in Core-Collapse Supernova Shock Revival," Phys. Rev. D **85**, 065008 (2012) [arXiv:1106.1167 [astro-ph.SR]].

[39] O. Pejcha, B. Dasgupta and T. A. Thompson, "Effect of Collective Neutrino Oscillations on the Neutrino Mechanism of Core-Collapse Supernovae," Mon. Not. Roy. Astron. Soc. **425** (2012) 1083 [arXiv:1106.5718 [astro-ph.HE]].

[40] H. Duan, A. Friedland, G. C. McLaughlin and R. Surman, "The influence of collective neutrino oscillations on a supernova r-process," J. Phys. G **38**, 035201 (2011) [arXiv:1012.0532 [astro-ph.SR]].

[41] E. Pllumbi, I. Tamborra, S. Wanajo, H. T. Janka and L. Hdepohl, "Impact of neutrino flavor oscillations on the neutrino-driven wind nucleosynthesis of an electron-capture supernova," Astrophys. J. **808**, no. 2, 188 (2015) [arXiv:1406.2596 [astro-ph.SR]].

[42] B. Dasgupta and J. F. Beacom, "Reconstruction of supernova ν_μ, ν_τ, anti-ν_μ, and anti-ν_τ neutrino spectra at scintillator detectors," Phys. Rev. D **83** (2011) 113006 [arXiv:1103.2768 [hep-ph]].

[43] P. D. Serpico, S. Chakraborty, T. Fischer, L. Hudepohl, H. T. Janka and A. Mirizzi, "Probing the neutrino mass hierarchy with the rise time of a supernova burst," Phys. Rev. D **85**, 085031 (2012) [arXiv:1111.4483 [astro-ph.SR]].

[44] E. Borriello, S. Chakraborty, A. Mirizzi, P. D. Serpico and I. Tamborra, "(Down-to-)Earth matter effect in supernova neutrinos," Phys. Rev. D **86**, 083004 (2012) [arXiv:1207.5049 [hep-ph]].

[45] J. F. Beacom and M. R. Vagins, "GADZOOKS! Antineutrino spectroscopy with large water Cherenkov detectors," Phys. Rev. Lett. **93**, 171101 (2004) [hep-ph/0309300].

[46] L. M. Magro, talk given at Particle Physics and Cosmology Workshop 2015, Center for Theoretical Underground Physics and Related Areas, Deadwood, June 30, 2015.

[47] C. Lunardini and I. Tamborra, "Diffuse supernova

neutrinos: oscillation effects, stellar cooling and progenitor mass dependence," JCAP **1207**, 012 (2012) [arXiv:1205.6292 [astro-ph.SR]].

[48] A. Banerjee, A. Dighe and G. Raffelt, "Linearized flavor-stability analysis of dense neutrino streams," Phys. Rev. D **84** (2011) 053013 [arXiv:1107.2308 [hep-ph]].

[49] L. Wolfenstein, "Neutrino Oscillations In Matter," Phys. Rev. D **17**, 2369 (1978); S.P. Mikheev and A.Yu. Smirnov, "Resonance Enhancement Of Oscillations In Matter And Solar Neutrino Spectroscopy," Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)].

[50] G. G. Raffelt and G. Sigl, "Self-induced decoherence in dense neutrino gases," Phys. Rev. D **75**, 083002 (2007) [hep-ph/0701182].

[51] A. Esteban-Pretel, S. Pastor, R. Tomàs, G. G. Raffelt and G. Sigl, "Decoherence in supernova neutrino transformations suppressed by deleptonization," Phys. Rev. D **76**, 125018 (2007) [arXiv:0706.2498 [astro-ph]].

[52] R. F. Sawyer, "The multi-angle instability in dense neutrino systems," Phys. Rev. D **79**, 105003 (2009) [arXiv:0803.4319 [astro-ph]].

[53] J. P. Boyd, "Chebyshev and Fourier Spectral Methods," Second edition, Dover, New York (2001).

[54] R. F. Sawyer, "Speed-up of neutrino transformations in a supernova environment," Phys. Rev. D **72**, 045003 (2005) [hep-ph/0503013].

[55] S. Chakraborty, T. Fischer, A. Mirizzi, N. Saviano and R. Tomàs, "No collective neutrino flavor conversions during the supernova accretion phase," Phys. Rev. Lett. **107**, 151101 (2011) [arXiv:1104.4031 [hep-ph]].

[56] S. Chakraborty, T. Fischer, A. Mirizzi, N. Saviano and R. Tomàs, "Analysis of matter suppression in collective neutrino oscillations during the supernova accretion phase," Phys. Rev. D **84**, 025002 (2011) [arXiv:1105.1130 [hep-ph]].

[57] S. Sarikas, G. G. Raffelt, L. Hudepohl and H. T. Janka, "Suppression of Self-Induced Flavor Conversion in the Supernova Accretion Phase," Phys. Rev. Lett. **108**, 061101 (2012) [arXiv:1109.3601 [astro-ph.SR]].

[58] S. Chakraborty, A. Mirizzi, N. Saviano and D. d. S. Seixas, "Suppression of the multi-azimuthal-angle instability in dense neutrino gas during supernova accretion phase," Phys. Rev. D **89**, no. 9, 093001 (2014) [arXiv:1402.1767 [hep-ph]].

[59] S. Chakraborty, G. Raffelt, H. T. Janka and B. Mueller, "Supernova deleptonization asymmetry: Impact on self-induced flavor conversion," arXiv:1412.0670 [hep-ph].